Evolutionary Selection of Zernike Moment Sets In Image Processing

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***Abstract*—In this paper, a new method that provides the ability of appropriate selection of useful Zernike moment sets is presented. Considering that Zernike moments of different orders carry sufficient information about the image, (low order describes coarse features of the image and high order the details), the need for a fixed number of appropriate Zernike moments, depending on the application, is understandable. Specifically, some fundamental demands in pattern recognition tasks must be satisfied, such as fixed, informative and discriminative features, suitable for a successful classification process. This target can be achieved by using an optimization tool, which searches in complex search spaces. In this paper a Genetic Algorithm (GA) is used, in order to find the suitable Zernike moment set, that yields to a near optimum classification result.**

Keywords – genetic algorithms; zernike moments; feature selection; pattern recognition

I. INTRODUCTION

It has been a long time since the first introduction of moments in image processing [1]. Zernike moments are a set of orthogonal moments that have complex kernel functions based on Zernike polynomials. They have been used successfully as a tool in image processing because of their ability to decompose an image into its high and low resolution components. Increasing the order of computed Zernike polynomials theoretically to infinity, full image representation can be achieved. Such representation that carries the whole image information is practically impossible. So there is a need of a method, which can decide the appropriate set of Zernike moments to be computed, in order to decrease the dimension of the feature vector (in a fixed number) without losing useful information, depending on the application. In this paper, the use of an evolutionary searching method (GA) is tried, in order to decide, which Zernike moment sets are suitable in a predefined pattern recognition application. For this reason a conventional neural classifier is used, to classify some patterns that are described by their near optimum Zernike moment feature vectors, which are obtained by the genetic algorithm stated in the previous processing stage.

II. ZERNIKE MOMENTS

Zernike introduced a set of complex polynomials, which form a complete orthogonal set over the interior of the unit circle $x^2+y^2=1$. These polynomials [2] have the form

$$V_{pq}(x,y)=V_{pq}(r,\vartheta)=R_{pq}(r)\exp(jq\vartheta) \qquad (1)$$

where p is non-negative integer, q is a non zero integer subject to constraints (i) p-$|q|$ being even, (ii) $|q|\leq p$, r is the length of vector from origin $(\bar{x},\bar{y})$ to pixel with coordinates (x,y), θ the angle between vector ρ and x axis in counter-clockwise direction, $R_{pq}(r)$ are the Zernike radial polynomials in (r,θ) polar coordinates defined as

$$R_{pq}(r)=\sum_{s=0}^{p-|q|/2}(-1)^s \cdot\frac{(p-s)\,!}{s!\left(\frac{p+|q|}{2}-s\right)!\left(\frac{p-|q|}{2}-s\right)!}r^{p-2s} \qquad (2)$$

Note that $R_{p,-q}(r)=R_{pq}(r)$

Zernike moment of order n with repetition m for a digital image with intensity function $f(x,y)$ that vanishes outside the unit disk is

$$Z_{pq}=\frac{p+1}{\pi}\sum_x\sum_y f(x,y)V^*_{pq}(r,\theta),\qquad x^2+y^2\leq 1 \qquad (3)$$

The rotation invariant property of these Zernike moments has been already analyzed [2]. These investigations led to the conclusion that the magnitudes of Zernike moments are invariant to any rotation of the image. Thus, the magnitudes of the resulted Zernike moments beyond a high order can be used for our experiments.

Because Zernike moments are only rotationally invariant, additional properties of translation and scale invariance must be given to these moments in some way. We can ensure these invariances by converting the absolute pixel coordinates [2].

According to (2) there are a lot of computations (factorials) that should be taken in account, in order to calculate the radial polynomials. For this reason many researchers have introduced methods for fast computation of Zernike moments [3]. Among these there is an efficient one [4] the well-known "q-recursive method". This method permits the evaluation of radial polynomials by using the following recursive equations,

$$R_{pq}(r) = r^{p} \quad for \quad p = q \tag{4}$$

$$R_{p(p-2)} = pR_{pp}(r) - (p-1)R_{(p-2)(p-2)}(r) \ for \ p - q = 2 \tag{5}$$

$$R_{p(p-4)}(r) = H_1 R_{pq}(r) + (H_2 + \frac{H_3}{r^2}) R_{p(q-2)}(r) \quad otherwise \tag{6}$$

where the coefficients H_1, H_2 and H_3 are given by

$$\begin{aligned} H_1 &= \frac{q(q-1)}{2} - qH_2 + \frac{H_3(p+q+2)(p-q)}{8} \\ H_2 &= \frac{H_3(p+q)(p-q+2)}{4(q-1)} + (q-2) \\ H_3 &= \frac{-4(q-2)(q-3)}{(p+q-2)(p-q+4)} \end{aligned} \tag{7}$$

Using this method, individual pth-order moments can be calculated without employing higher or lower order moments. This allows the entire set of Zernike moments to be derived independently. This feature is useful for applications where only selected orders of Zernike moments are needed as pattern features and fast computation (fewer multiplications) is required.

The proposed method is a mechanism to select specific orders of Zernike moments, utilizing the advantages derived by using q-recursive method as a way of computing each moment.

In each generation of the genetic process, where new offsprings are produced, the q-recursive method is applied to compute the respective moment order, which is represented by the value of the products.

III. GENETIC ALGORITHMS

Genetic Algorithms (GAs) have played a major role in many applications of the engineering science. As mentioned above, GAs constitute a powerful tool for optimization as well as other tasks. A simple genetic algorithm is a stochastic method that performs searching in wide search spaces and depends on some probability values. For these reasons it has the ability to converge to the global minimum or maximum, depending on the specific application, and to skip possible local minima or maxima, respectively.

The main idea in which GAs are based, was first inspired by J. Holland [5]. He tried to find a method to mimic the evolutionary process that characterizes the evolution of living organisms. This theory is based on the mechanism qualified by the survival of the fittest individuals over a population. In fact, there are some specific procedures taking place until the predominance of the fittest individual.

In the sequel, terminology in the field of genetic methods for optimization and searching purposes is given according to [6]:

- *Individual* is a solution of a problem satisfying the constraints and demands of the system in which it belongs.
- *Population* is a set of candidate solutions of the problem, which contains the final solution.
- *Fitness* is a real number value that characterizes any solution and indicates how proper is the solution for the problem under consideration.
- *Selection* is an operator applied to the current population, in a manner similar to the one of natural selection found in biological systems. The fitter individuals are promoted to the next population and poorer individuals are discarded.
- *Crossover* is the second operator that follows the previous one. This operator allows solutions to exchange information, in such a way that the living organisms use in order to reproduce themselves. Specifically two solutions are selected to exchange their sub-strings from a single point and after, according to a predefined probability P_c. The resulting offsprings carry some information from their parents. In this way new individuals are produced and new candidate solutions are tested in order to find the one that satisfies the appropriate objective.
- *Mutation* is the third operator that can be applied to an individual. According to this operation its single bit of an individual binary string can be flipped with respect to a predefined probability P_m.
- *Elitism* is the procedure according to which, the fittest individual of each generation is ensured to be maintained in the next generation.

After these operators have been applied to the current population, a new population is formed and the generational counter is increased by one. This process will continue until a predefined number of generations are attained or some form of convergence criterion is met.

In Fig.1, a block diagram of a simple evolutionary selection (GA) process, is depicted. This conventional genetic algorithm consists of a preprocessing stage, in which the suitable calibration is performed, followed by the main searching procedure. During initialization, a set of parameters (genetic operators, initial population, etc.) must be set to obtain the desired operation of the algorithm.

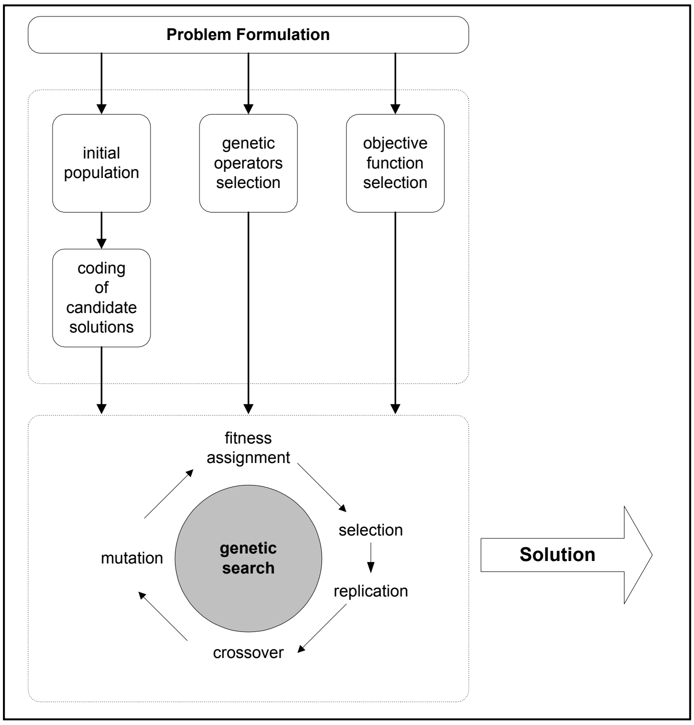


Figure 1. Evolutionary searching Method

IV. THE PROPOSED METHOD

It was already mentioned in section II that the power of Zernike moments is the description of an image in multiple resolutions. This derives from the computation of different Zernike moment orders. Theoretically, by computing the Zernike polynomials the whole image information can be captured. However, this scenario is impossible, for practical tasks. Additionally there is a major requirement in pattern recognition problems to use fixed feature vectors. Moreover, these vectors should be as small as possible maintaining the useful, discriminative image information.

The proposed method implements a two-stage classification procedure, with a genetic algorithm in the first one and a typical neural classifier in the second, as illustrated in Fig. 3.

To be more specific, let's define a pattern classification task. Consider some patterns, which are to be classified with the use of a classifier, consisting of a multilayer perceptron neural network having fixed number of inputs, a hidden layer with some neurons, and an output layer having the same number of outputs as the patterns.

Choosing Zernike moments as discriminative features used as inputs in the classifier, a major question comes up related to the orders of the Zernike moments, which should be used, while the number of moments is kept fixed. This question is tried to be answered by a genetic algorithm, which searches, in a stochastic way, into the search space of Zernike moments for the appropriate Zernike set (fixed number of Zernike moments to be inserted in the classifier) of the current patterns. In each generation the resulted Zernike moment set is guided to the classifier inputs and the training process of the neural classifier is started.

After the completion of the classifier training, the same orders of Zernike moments for each pattern are tested to the classifier, which operates at this moment in generalized mode, and a fitness value is assigned to each candidate Zernike moment set of the patterns. Extra attention is needed in the selection of the appropriate fitness function, which the genetic algorithm is called to minimize or maximize.

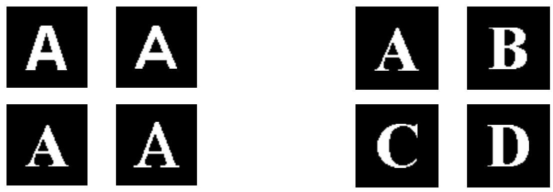
In our experiments the MSE of the classification procedure is used, as a fitness function. Therefore the genetic algorithm comes to solve a minimization problem, finding this Zernike moment set for the patterns that gives the near optimum solution (minimum MSE) of the problem.

$$MSE = \frac{1}{N}\sum_{i=1}^{N}\left(d_i - y_i\right)^2 \qquad (8)$$

where the term $(d_i - y_i)$ represents the generalized classification error of the i^{th} output of the classifier when the proposed feature vector of a tested pattern is presented.

V. SIMULATION RESULTS

In order to investigate the performance of the proposed method two sets of experiments have been considered. In the first one, a classification problem of similar letters in different fonts (Fig. 2a) has been defined, while in the second one the patterns were decided to be absolutely different (Fig.2b).



(a) (b)

Figure 2. (a) the same letter in different fonts, (b) different letters

The configuration parameters of the GA that have been used are, population size of 30 candidate solutions, binary representation of length 6, single point crossover with Pc=0.4, mutation with Pm=0.01, wheel roulette selection method, no elitism and generation number of 100.

Also a typical multilayer perceptron with (7-7-4) structure has been used as a classifier for the classification of each four element set in each experiment.

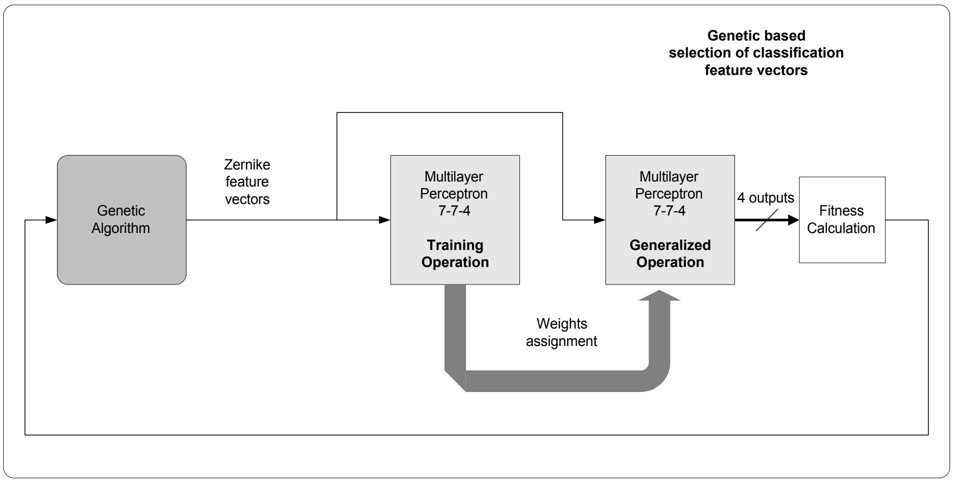
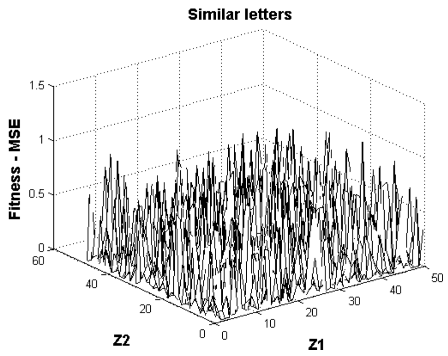
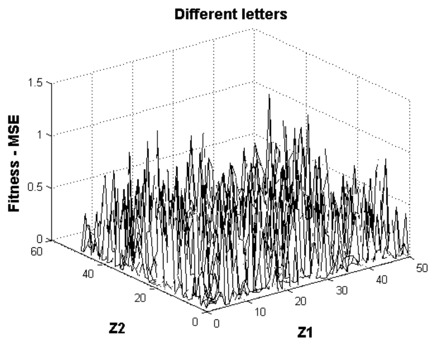


Figure 3. Block diagram of the proposed selection method



(a)



(b)

Figure 4. Search spaces of each experiment

In Fig. 4 the search spaces for the two experiments above are depicted in the case of two-element feature vector. As it can be seen, these spaces are quite complex with a lot of local minima and maxima but there is a unique minimum that correspondes to a pair of (Z1,Z2) values.

The resulted moment sets for the two experiments are: [Z_{4844}, Z_{5654}, Z_{3838}, Z_{1515}, Z_{3430}, Z_{2828}, Z_{4014}] for the 1st experiment and [Z_{10}, Z_{4309}, Z_{5614}, Z_{4545}, Z_{3333}, Z_{2424}, Z_{4935}] for the 2nd, where the two first indices corresponds to the moment's order (up to 63 order) and the last two to the moment's repetition.

VI. CONCLUSIONS

A new method, which has the ability to derive an appropriate Zernike moment set as feature vector in a classification task, has been presented. Using the proposed mechanism it is ensured that these features contain image information that is needed to optimally classify the tested patterns. For similar patterns the discriminative information is described in high Zernike moments orders while for different objects in lower ones. Thus dependent on application, a suitable Zernike moments set that guarantees low generalized classification error can be derived.

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